I-6 ELECTROMAGNETIC RESONANCES OF FREE DIELECTRIC SPHERES

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The last few years have seen the publication of several studies of electromagnetic resonances of free dielectric samples. This is explained by the high Q factor that can be obtained with a little space-factor, when using high dielectric constant materials and a low dissipation factor (tg δ). The resonators used lend themselves to a number of different applications. However their use requires a good knowledge of their spectra and field configuration.

We have made a systematic study of the resonances of an isotropic spherical dielectric sample, supposed without loss, placed in an infinite medium. To our knowledge, this problem, up until the present time, has only been approximately resolved, the results obtained only being valid for a dielectric constant ϵ which was sufficiently large[1].

The dielectric sample created in its proximity a concentration of semi-stationary electromagnetic energy. By using the Bromwich's method, we have resolved Maxwell's equations taking into account the boundary conditions – zero energy at infinity. This leads to a characteristic equation which is both complex and transcendant; its resolution in ω gives the free oscillation frequencies of the system and their relaxation times.

To help us solve the equation we used a computer with a chequering method followed by one of double iteration. By using the spherical co-ordinates we have been able to class the modes as following: TE_{nmr} and TM_{nmr} where the three indices nmr correspond respectively to the order of Bessel's function, the order of Legendre's polynome and the order of the root of the characteristic equation.

With the only restruction $m \leq n$ all the TE modes were found to exist whatever the value of ϵ . This however was not the case for the TM modes which only existed when the r values were sufficiently small, the cut-off being a function of ϵ .

As an example, we give the frequency and Q factor, where ϵ varies between 2 and 100 (Fig. 1 and 2), for the first seven modes.

The Q factor thus calculated represents the relation between the stored energy of the resonant system and the radiated energy per cycle. This is the maximum theoretical Q obtained in the case of non-loss materials. This value is of great interest since it indicates the possibility of coupling the mode to the exterior medium. A theoretical value for Q which is too low indicates a stored energy which is likewise too low thus preventing the mode from being observed. A value for Q which is too large however ($Q \gg 1/\text{tg}\,\delta$) indicates that the dielectric losses can no longer be neglected since they are greater in number in comparison with the radiation; such a mode cannot be observed either.

In an approximation of negligible radiation in comparison with the stored energy, we have studied the force lines of the first seven modes, on the interior of the dielectric as well as the exterior.

For the TE modes the force lines of the electric field are on a sphere. In the case where the index m is zero the modes are axially symetric, and the force lines of the electric field are circles of the axis $\theta = 0$. The force lines of the magnetic field are situated in one of the meridian planes.

Figures 3 and 4 represent, as an example, the force lines of the magnetic field in a meridian plane for the TE_{101} and TE_{301} modes (for a dielectric sphere of constant $\epsilon = 14$).

In the TM modes, the nature of the force lines remains the same but the role of the H and E fields is permutated.

The modes of a higher order can be classed as volume modes and surface modes according to whether r >> n or n >> r; in the latter case the energy is concentrated in the proximity of the separation surface.

These modes have been experimentally studied using a set-up as shown in Fig. 5 at X and Ku bands. Fig. 6 shows the results obtained with a sphere 3.9 mm in diameter and where $\epsilon = 86$. Verification is therefore excellent. These modes lend themselves to interesting applications. The strong concentration of energy given by these modes has permitted us to realize a power limiter using the TE_{101} mode of the sphere of YIG[2].

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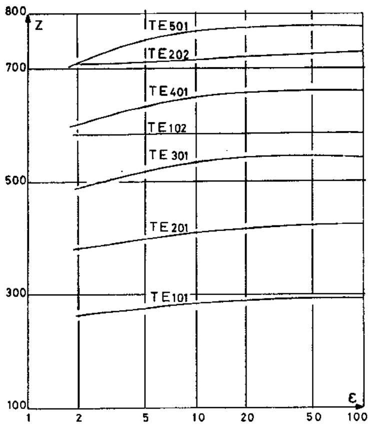

FIG. 1 $Z = F_{Gc/s} D_{mm} \sqrt{\epsilon}$

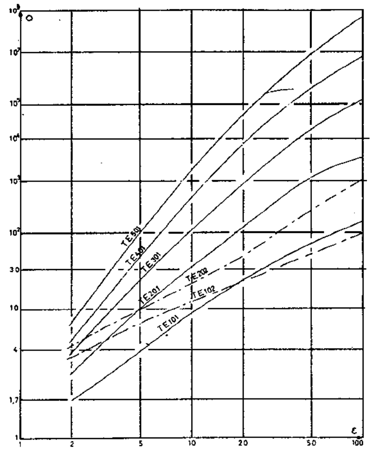

FIG. 2

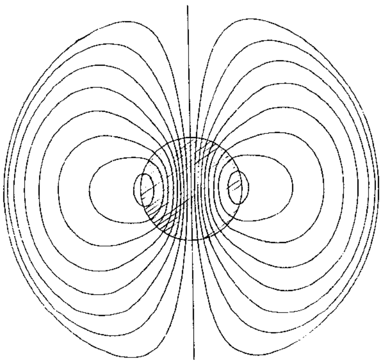

FIG. 3 MAGNETIC FORCE LINES OF THE TE_{101} MODE IN A DIELECTRIC SPHERE (ϵ=14)

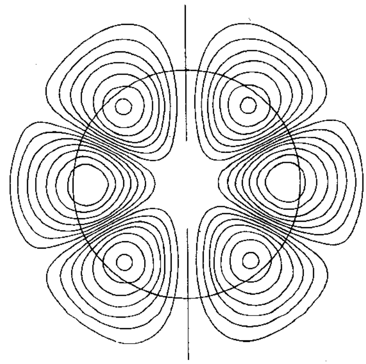

FIG. 4 MAGNETIC FORCE LINES OF THE TE_{301} MODE IN A DIELECTRIC SPHERE (ϵ=14)

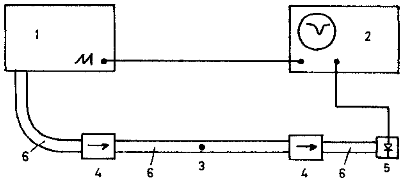

FIG. 5 EXPERIMENTAL SET-UP IN ORDER TO MEASURE THE NATURAL FREQUENCY OF A DIMENSIONAL RESONANCE

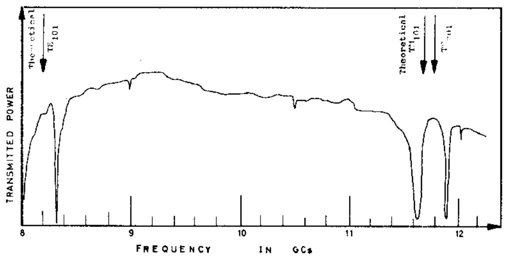

FIG. 6 RECORD OF THE RESONANCES OF A DIELECTRIC SPHERE AT X BAND ($\epsilon = 86$; $\phi = 3.9$ mm)